Chapter 1

INTRODUCTION

The research described here was undertaken as a 2-year investigation of two sets of related questions:

1. How does teacher understanding of rational number, quantity, and proportional reasoning influence the manner in which teachers teach? That is, what changes and shifts, both subtle and overt, can be noted in the way these topics are treated by the teacher as the teacher becomes more familiar with the mathematics involved and comes to understand better how students learn this content? After teachers have opportunities for study and reflection, how does a teacher's decision making change? How does a better understanding of the mathematics and the ways in which students come to learn this mathematics influence teachers' views about what it means to teach mathematics?
2. As teachers' understanding of rational number, quantity, and proportional reasoning develops and as teachers become more aware of how students learn this mathematics, how is their students' learning enhanced? How does student understanding change over the course of a year?

ORIGIN AND BACKGROUND OF THE STUDY

This research was carried out as a project of the Teaching and Learning Rational Numbers and Quantities Working Group, one of the seven working groups of the National Center for Research in Mathematics Education (1990–1995). This Working Group was composed of members representing many different orientations and theoretical perspectives, but all members had research interests that focused, at least in part, on various aspects of the multiplicative conceptual field. The orientations

and perspectives of the individual members guided them to undertake research that varied widely in the types of questions addressed, the methodologies used, the interpretation of data, and the manner of disseminating and sharing results. Overarching these differences were commonalities of interest and agreement that included, at a minimum, consensus on the complexity of the field under study, the need to come to a fuller understanding of these complexities and how they should be addressed in a classroom setting, and the need to provide teachers with stronger pedagogical understandings of the mathematical content in question. The initial meeting of this Working Group was held in May of 1991. Four local middle-grades teachers also attended the meeting. One of the outcomes of this meeting was the recommendation that research be undertaken to investigate the relationship between middle-grades teachers' knowledge of mathematics and their instructional practices within the area of the multiplicative-structures domain.

This recommendation was formulated into the research questions stated at the beginning of this section and was undertaken by some of the Working Group members located at San Diego State University. A 1-year preparatory investigation into these questions focused on the four teachers who had attended the initial Working Group conference. These teachers were well known to the researchers from their participation in past projects and in graduate coursework, from their excellence in teaching as noted by observation and through awards received, and from their leadership in mathematics education in San Diego. The purpose of this initial year of work was not only to explore the research questions with excellent teachers, but also to develop both the conceptual framework and the procedural plan for our long-term study. Results of our work with these teachers is reported elsewhere (Flores, Sowder, Philipp, & Schappelle, 1995; Philipp, Flores, Sowder, & Schappelle, 1994; Philipp, Sowder, & Flores, 1992; Philipp, Sowder, Flores, & Schappelle, 1995; Sowder, Philipp, Flores, & Schappelle, 1995), and is not further discussed here. These four teachers assisted us in designing the research project represented in this monograph.

AN OVERVIEW OF CONTENTS

In the chapter following this introduction we present the conceptual framework for this investigation. This framework is a refinement of the original framework (J. Sowder, 1992), prepared at the beginning of our planning year. By attempting to define the content domain in terms of what teachers need to know, we expected the boundaries of that domain to change during the course of our work. In this case, several researchers associated with the Working Group presented to the teachers seminars

in which they described research-based principles and projects related to content topics relevant to this project, to be used to guide classroom instruction. The seminars and associated papers, together with further discussions of the presentations during other seminars, came to define, for the teachers, the bulk of the content domain focused on by this project. The revised conceptual framework reflects this influence. (The papers, by Armstrong and Bezuk; Harel; Kieren; Lamon; Mack; J. Sowder; L. Sowder; and P. Thompson, are available in another project publication, edited by J. Sowder and Schappelle, 1995.)

The third chapter provides details on the methodology of our study. We describe there the selection of the teachers, the content of the seminars with the teachers over the 2-year period, and our data collection, which consisted of interviews of the teachers, tests of the content knowledge of the teachers, protocols of seminar interactions, many classroom observations throughout the 2-year period, and tests of student understanding of the mathematics being addressed by this study. We also discuss the manner in which we handled our concerns about matters of validity, reliability, and ethics. Finally, we attempt to articulate our own assumptions and biases, here and throughout the chapters, insofar as they might have an effect on the investigation undertaken and described in this report.

Chapters 4 through 10 directly address the two research questions that guided our work and that appear at the beginning of this chapter. In chapter 4 we explore the effects of our seminars with the teachers on their development of both mathematical understanding and understanding of how students learn this mathematics. Chapters 5 through 9 are case studies of five of the teachers associated with the project. Chapter 10 contains an analysis of the student data we collected.

There were results that we had either not anticipated or that we had chosen not to investigate, but which, in retrospect, we deemed worthy of discussion. These results are discussed in chapter 11. We take the opportunity in this chapter to reflect on the effects this project had on us as researchers and as teachers and to make some recommendations about teacher professional development, both preservice and inservice.

THE CASE STUDY APPROACH

Case study is a method of choice when the purpose of a study is to examine the interaction of significant factors associated with a phenomenon. In this study, many factors influenced teacher change over the 2-year period, and any attempt to examine these factors and place them in perspective demanded a multileveled approach both to the

kinds of data collected and to the data interpretation. An interpretive case study (as opposed to a descriptive or an evaluative case study) depends on rich, thick description to develop conceptual categories and to illustrate and support (or challenge) assumptions held at the beginning of the study (Merriam, 1988). This approach appeared to us to be an appropriate way to analyze and present data we collected relative to the teachers. We assumed, in particular, that our intervention would positively influence instructional practices and student learning, that the teachers would themselves come to understand the mathematics better and would effect deeper conceptual understanding on the part of their students. Most of the data we collected were qualitative in nature: seminar protocols, interview protocols, and classroom-observation data. This type of data is regarded as more likely than quantitative data to yield insight into changes that occur (Merriam, 1988). Some of our data were quantitative (a content understanding test given to the teachers; tests of fraction understanding and ability to reason proportionally given to students); but these data were used primarily to support the qualitative data analysis.

Our choice of composition of this case-study approach also deserves comment. Among written forms of multiple-case studies, there exist the possibilities of presenting cases (teachers, in this study) singly as individual case narratives or focusing on cross-case issues with information on individual cases dispersed throughout the chapters (Yin, 1984). The second format was very attractive to us because it allowed us more protection of the identities of the teachers involved in the study. However, this approach simply did not work for us. When we attempted to follow the second format, we felt that we lost too much of the information needed for interpretive analysis, and we found ourselves returning in our writing to a focus on the individual case narratives. We therefore decided on the individual case-study format. Individual case studies have been used successfully by other researchers interested in teacher change (or lack of it). These include, for example, the case studies by researchers at the National Center for the Study of Teacher Learning intended to document the effects of policy on elementary mathematics teaching (Ball, 1990; Cohen, 1990; Peterson, 1990; Wiemers, 1990; Wilson, 1990) and case studies by Schifter and Fosnot (1993) in their investigation of teacher change resulting from SummerMath.

Stake (1994) has said, "A case study is both the process of learning about the case and the product of our learning" (p. 237). Developing case studies of these teachers has been a long and fruitful process for us, and we hope that the product we provide here adequately portrays what we have learned.