Chapter 1

The Several Senses of "Analysis" in Aristotle

The title of Aristotle's work, *Analytics* (*Analutikōn*), comes from the Greek word *analutos*, and its verb, *analuein*. This chapter is devoted to a clarification of these terms. Following a cursory etymology of the terms, I will examine the possibility that Aristotle derived his notion of analysis from his great teacher, Plato. I will then undertake a comprehensive survey of the different ways in which Aristotle used *analuein* and its cognates. I shall argue that (1) there are several distinguishable sets of meanings for these terms in Aristotle's *corpus*; (2) while "decompose"—the most prevalent connotation of "analyse" in the modern period—is among Aristotle's meanings, it is neither the sole meaning nor the principal meaning nor the meaning which best characterizes the work, *Analytics*; (3) the meaning that does best characterize the *Analytics* is one that Aristotle most likely derived from the ancient practice of geometrical analysis; and that (4) this meaning is closely related to Aristotle's meaning of *epistēmē* (science) as knowing the "reasoned fact."[1] Thus, we might call analysis in this sense the process of transition from "mere fact" to "reasoned fact." Analysis, then, is the way to or the process of finding the "reasoned fact."

A. A BRIEF ETYMOLOGY

The ancient Greek term *analutos* meant "soluble." The verb *analuein* has been translated "to solve," "to resolve," and even occasionally "to dissolve." *Analuein* itself comes from the verb *luein*, "to loose," and the prefix *ana*, "up." Clearly the image suggested by *analuein* is one of the "loosing up" of something compact into its constituents, as when a solid is dissolved in water. "Analysis," therefore, is concerned with "loosing up," with "solubility," and with "solution." Moreover, just as in modern English one may speak both of a liquid solution and the solution of a puzzle or problem, so also these connotations are found among the usages of *analusis* in ancient Greek.

Perhaps none of this sounds unusual to a modern reader who is used to thinking of analysis as the reduction of a whole into atomic parts, either through the techniques involving the "supercolliders" of particle physics, the apparatuses of analytic chemistry, or dissection techniques of experimental anatomy. However, there is no real reason why the ancient Greek idea of "loosing up" should be the same as the modern notion of "reducing" or "breaking up."[2] This is a specifically modern way of thinking that was perhaps initiated by Francis Bacon when he wrote:

> Now what the sciences stand in need of is a form of induction which shall *analyze experience and take it to pieces,* and by a due process of exclusion and rejection lead to an inevitable conclusion.[3]

We find this same idea of analysis in the *Opticks* of Isaac Newton, who gave to it an even greater impetus when he wrote:

> As in mathematics, so in natural philosophy, the investigation of difficult things by the method of analysis, ought ever to precede the method of composition. This analysis consists in making experiments and observations, and in drawing general conclusions from them by induction, and admitting of no objections against the conclusions, but such as are taken from experiments or certain other truths. For hypotheses are not to be regarded in experimental philosophy . . . By this way of analysis *we shall proceed from compounds to ingredients*, and from motions to the forces producing them; and in general from effects to their causes, and from particular causes to more general ones, till the argument end in the most general. This is the method of analysis.[4]

This modern, Baconian notion of analysis was gradually assimilated into the natural sciences, perhaps last of all into biological science. According to historian William Coleman, it was in 1798 that Philipe Pinel urged "that medicine should adopt, as had the other sciences, the method of philosophical 'analysis'."[5] Pinel's suggestion was pursued by Xavier Bichat, who developed dissection practices to distinguish different organic tissues and who held, as Coleman narrates it,

> If we wish to know . . . the "properties of life" of [an] organ, we must *"decompose it," that is, only if we "analyse [it] with rigor"* can we know "its intimate structure."[6]

It is this Baconian way of thinking about analysis as "decomposition" that holistic thinkers such as Henri Bergson have continually railed against.[7]

However, it is not necessary that "loosing up" connote either "reducing to independent atomic particles," or "decomposing" a whole, living organism by cutting it into non-living parts. Aristotle, for one, certainly did not think according to philosophies of science which arose two millennia after his death. If we are to understand Aristotle in his own right, it is essential that we do not read this reductionistic interpretation of analysis back into his writings.[8] Instead, we need to go back behind these inherited associations to his original context.

If we accept the suggestion that Aristotle's meaning of "analysis" is not the same as the reductionistic modern meaning, we are still left with the question, Just what was his meaning? Furthermore, what does analysis and its connotations have to do with science? What sort of "loosing up" into constituents is Aristotle's *Analytics* concerned with?

B. ANALYSIS IN PLATO?

In this section I wish to make a fairly simple point: although there is a long-standing tradition ascribing the origin of analysis to Plato, this offers virtually no assistance in determining how Aristotle understood analysis. Therefore we are thrown back upon Aristotle's own writings in order to arrive at that determination. However, because the possible relationship between ancient geometrical analysis and Plato's dialectic has received extensive scholarly discussion, it will require a lengthy discussion to establish my point.

Plato has been widely held to be the originator of the "analytic method," primarily based upon a pair of passages from Proclus's *Commentary* on Euclid's *Elements*:[9]

> Eudoxus of Cnidos, a little younger than Leon, on terms of friendship with Plato's circle, enlarged for the first time the number of so-called general theorems, joined three more to the three mean proportionals and continued the researches on the section, begun by Plato, making use of analysis.[10]

> Nevertheless, there are certain methods that have been handed down, the best being the method of analysis, which traces the desired result back to an acknowledged principle. Plato, it is said, taught this method to Leodamas, who also is reported to have made many discoveries in geometry by means of it.[11]

Thus, turning to Aristotle's teacher, Plato, would seem to be an obvious first step toward acquiring some hint as to the sources of Aristotle's understanding of *analusis*.

However, the passages from Proclus do not unambiguously attribute either the origination or even the use of analysis to Plato. Only preliminary researches "on the section" are explicitly attributed to Plato himself. The phrase, "making use of analysis," can be read more plausibly as referring to Eudoxus than to Plato. Likewise Proclus is notably cautious in reporting that Plato was only "said" to have taught the method of analysis to Leodamas, and the most likely reading is that the "discoveries" by means of analysis are being attributed to Leodamas, not to Plato. There is also a related passage in Diogenes Laertius,[12] but this only states that Plato explained the method of analysis to Leodamas of Thasos, not that he originated the method.

Another source of the view that Plato originated the method of analysis is his discussion of mathematics in the *Republic*, especially in Books VI and VII. There Plato contrasts the method of "those who occupy themselves with geometries and calculations" with his philosophical method of dialectic. The former take their "hypotheses" for granted and, not thinking it "worthwhile to give any account of them to themselves or others," they proceed "downward" to deduce results. The dialectician, on the other hand, regards the hypotheses as "really hypotheses—as steppingstones and springboards" and moves "upward" towards intelligible principles "free from hypothesis."[13] There has been considerable discussion of the relationship between the "upward" and "downward" movements in ancient geometry on the one hand, and in Plato's dialectic on the other. Norman Gulley indicates that some ancient commentators believed that Plato originated a very general philosophical analysis (dialectic), which was subsequently applied to a specific field such as geometry.[14] Among contemporary scholars, however, a general consensus has emerged that geometrical analysis certainly predated Plato's philosophical reflections upon it.[15]

Even so, there has been considerable disagreement as to the exact nature of geometrical analysis, and of how it influenced Plato's (as well as Aristotle's) thought. F. M. Cornford claimed that modern historians of mathematics (including Thomas Heath) had taken a description of analysis by Pappus[16] and had "made nonsense of much of it by misunderstanding the phrase '*the succession of sequent steps*' (*tōn hexēs akolouthōn*) as meaning logical 'consequences', as if it were *ta sumbainonta*."[17] Cornford argues, instead, that the upward movement of geometrical analysis involves "the divination of a premise that must be true if the required premise is to follow" (p. 40), an intuition (p. 47) or direct perception "without discursive argument that a prior condition must be satisfied" (p. 43). He goes on to argue that this act of non-deductive upward movement in geometrical analysis *is* "the mental experience" Plato called *noesis* "in one of its senses" (pp. 43, 48), the act which is the ultimate objective of dialectic. On the other hand, synthesis, the "downward" movement, was indeed a matter of logical deduction from principles arrived at by analysis, and as such was the basis for one of the senses of Plato's notion of *dianoia*.

In short order, Richard Robinson defended the honor of the historians of Greek mathematics who were "at one" concerning the method of geometrical analysis, namely, that analysis began with a proposition to be proven, and upwardly *deduced* a series of consequences.[18] Once a proposition independently known to be true is reached, the deductive sequence is reversed, constituting the downward synthesis. He criticized Cornford's interpretation, noting that unless each proposition in the ascending and descending series were convertible, the two-part method would not work. He then went on to claim that all ancient descriptions of geometrical analysis either favored this reading, or were too vague to be decisive (pp. 465–67). In support of his position, Robinson called upon a "vitally important set of texts" (p. 469) which Cornford had overlooked; namely, actual mathematical examples in ancient geometrical treatises themselves. Robinson examined one such example (using the gloss on Euclid's *Elements* Proposition XIII.1), arguing that both the analysis and the synthesis in that gloss relied exclusively upon equations and their transformations, meaning that each proposition in both the upward and downward sequences is convertible. However, he rested his case upon this lone example, and was forced to attempt explaining away a couple of difficulties even with it.[19]

More recently Kenneth Sayre has attempted to develop a line of argumentation similar to Robinson's, and to apply it to Plato. He argues that Plato's "method of hypothesis"—which is a method of collection and division[20]—not only is analytical method, but is indeed derived from ancient geometrical analysis.[21] According to Sayre, this method is formulated in *Phaedo* 100A and elaborated in the *Theaetetus* and the *Sophist*. However, the arguments of both Robinson and Sayre rely heavily upon a questionable assumption; namely, that ancient geometrical analysis always relied upon "convertible" propositions, of which equations are prototypical. Thus, according to Sayre,

> The technique [for geometrical analysis] was to treat the proposition to be demonstrated as an hypothesis, and then draw consequences from it, and further consequences from these, and so forth, until a proposition was reached which was already accepted as true, or which was recognized as independently demonstrable. This in itself, of course, does not amount to a proof of the proposition with which the problem originated. But it is a feature of geometry, not shared by logic either now or then, that it deals for the most part with assertions of equality in which equals can be added to or divided by equals, and in which any component term could be replaced by an equivalent term, *without change in truth value* of the original equations . . . Thus the propositions $AB + BC = 2CD$ and $AB = 2CD - BC$ are mutually deducible or *convertible*.[22]

While Sayre's argument depends in an integral way upon the claim that geometry deals for the most part with assertions of *equality*, Robinson's point is more general—that analysis has to do with convertible *propositions*, using a single illustration involving equations. Both, however, are vulnerable to the more nuanced cases developed by Gulley, Jakko Hintikka and Unto Remes, W. R. Knorr, and M. S. Mahoney.[23] By their exacting examinations of a comprehensive range of passages from ancient writers, these scholars build a compelling case for forms of geometrical analysis which do not presuppose convertible premises. Gulley in particular, while conceding a modified form of Robinson's thesis—i.e., that at least some instances of analysis were deductive, relying upon convertible premises—argues that this cannot be the whole story.[24] From his conclusion that there were both deductive and non-deductive forms of geometrical analysis, Gulley staked out a third line of interpretation in order to resolve the disputes over Pappus's description; namely, that "Pappus is repeating two different formulations of the method [of analysis], one describing it as an upward movement to prior assumptions from which an initial assumption follows . . . , the other as a downward [deductive] movement from an initial assumption."[25]

Other scholars are at variance with Gulley's proposed solution. Mahoney's and Knorr's interpretations of the Pappus passage, while admitting the occurrence of some non-convertible instances, tend to view convertibility as the prevailing situation in mathematical research.[26] Acording to them, the non-convertible situations are dealt with by adding a *diorismos* which "supplies the conditions under which an originally non-reversible step in an analysis may be made reversible."[27] Hintikka and Remes, on the other hand, draw explicit attention to what they see as a weakness in Gulley's solution,[28] and propose instead a four-part analysis-and-synthesis method in order to reconcile this and other difficulties.[29] A thorough discussion of the proposals of Hintikka and Remes would take us too far afield here. Suffice it to say that, while they go very far in invoking deductive structures in their construal of the ancient analytical method,[30] they do not completely rule out non-deductive or "unpredictable" procedures.

In support of the contention that there were non-deductive forms of analysis, I wish to add a further argument. Robinson and Sayre are correct in saying that "deductive" analysis is possible in the case of convertible propositions—and especially equations—but are incorrect in the prominence they claim that either equations or convertible propositions held in ancient geometry. It is true that complementary axioms (e.g., the axiom "equals added to equals yield equals" is complementary to "equals subtracted from equals yield equals") permit a reversal of the order of demonstrations when equations are involved, but this is primarily true of ancient arithmetic rather than ancient geometry.[31] Unlike arithmetic, geometrical propositions in general do not have the form of convertible propositions (a fact which Robinson obscures in the way he rephrases the argument of Euclid's Proposition XIII.1). For this very reason, even when geometrical equality is asserted, reliance upon simple complementary operations—such as equals added to or subtracted from equals—is insufficient to the task of finding a demonstration of the proposition in question.

To be specific, ancient geometrical propositions do not in general have the form "$X=Y$." Rather, in virtually every case involving equalities, they have either the form "If Z is such-and-such a figure, then $X=Y$," or "If $X=Y$, then Z is such-and-such a figure." This is true in such fundamental propositions from Euclid's *Elements* as the Pythagorean theorem (Proposition I.47: "In right-angled triangles the square on the hypotenuse is equal to the sum of the squares on the legs.") and Proposition I.27. ("If a straight line falling upon two straight lines make the alternate angles equal to one another, the straight lines will be parallel to one another.")[32] Both propositions involve equalities, but neither is an equation or any other kind of a convertible proposition. Nor can one apply a complementary operation (e.g., equals added to equals) to these propositions as such. Nor does either of these two propositions convert logically; the converse of each (Propositions I.48 and I.29, respectively) must be proven independently. Moreover, the proof of I.48 is not some simple inversion of the order of the demonstration of I.47, even though the "givens" and the "to be showns" are exactly reversed. Still less is the proof of I.29 a simple inversion of the order of the demonstration of I.27; I.27 is proven *per impossibile*, whereas I.29 is proven directly.[33] The respective demonstrations of the converses are quite distinct, so that knowledge of one is of little help in the analytic search for the proof of the other.[34]

Hence, the sort of deductive analysis Robinson and Sayre attribute to ancient geometry—and by extension, to Plato—is only useful when it is possible to reverse the order of demonstration. In turn, this is possible only when the propositions used in the demonstration are either convertible[35] or involve complementary axioms such as equals added to or subtracted from equals. Such demonstrations form an exceedingly limited class.[36]

In general then, there is compelling evidence that a non-deductive form of analysis pre-dated Plato and that he knew of it. It is also likely that, as Cornford contended, this non-deductive form of analysis had some impact upon the way Plato conceived the "upward" movement of his *dialectical* method. But does this mean that Plato used the *analytical* method, or that Aristotle's analytical method was derived from Plato? I do not think we have sufficient evidence for a definitive answer in either case. For one thing, it is quite significant that neither the term *analusis* nor any of its cognates appear anywhere in Plato's extant writings,[37] and one is forced to wonder why the alleged originator of so remarkable a method would omit any mention of it from his writings.[38] (This is in stark contrast with Aristotle, who uses the term *analysis* or one of its cognates close to forty times in his corpus.) For another thing, to use the "upward movement" metaphor in drawing a similarity between geometrical analysis and Plato's dialectic does not really tell us very much, especially once the possibility of a non-deductive analysis is admitted.[39] At the very least, these difficulties make it impossible to directly determine what Plato might have understood by *analusis*, let alone what Aristotle might have drawn from him.

Therefore, with regard to the question of how Plato's understanding of ancient geometrical analysis may have influenced Aristotle, we may summarize as follows. First, based on the passages from Diogenes Laertius and Proclus, as well as the acute interest Plato is known to have taken in contemporary mathematical developments, it is highly likely that Plato knew about the method of geometrical analysis. Second, however, there is no unequivocal source which establishes Plato as the originator of this method. It is more likely that the method originated earlier in the practices of mathematicians[40] and subsequently came to the attention of Plato and his Academy. Third, it is highly likely that these earlier practices included a non-deductive, as well as a deductive form of analysis. Fourth, while it may be the case that the method of geometrical analysis influenced Plato's philosophy, and even his "method of hypothesis," there are difficulties with the thesis that Plato adapted a purely deductive geometrical analysis heavily dependent upon the use of convertible propositions to philosophical purposes.

Finally, what of the relationship between ancient geometrical analysis and Aristotle's work? Although an attempt to answer this question will occupy much of the remainder of this book, it is at least worth emphasizing in the present context that Aristotle considered analysis to indeed include a broader, non-deductive aspect. For one thing, he says, in the *Posterior Analytics*:

> If it were impossible to prove truth from falsehood, it would be easy to make an analysis [*to analuein*]; for they [i.e., conclusion and premise] would convert from necessity . . . (In mathematics things convert more [often] because they assume nothing accidental—in this too they differ from dialectics [*dialogois*]—but only definitions.) (I.12 78a6–13)[41]

Gulley rightly notes that this passage reveals Aristotle's awareness of a reversible form of analysis which employs convertible propositions.[42] However, I believe that, in addition, it provides further evidence of non-deductive analysis as well. If analysis *always* dealt solely with convertible premises, it would be dealing with a select set of propositions in which it would indeed be impossible to prove truth from falsehood. Moreover, this would make analysis "easy," since all analyses would be mere matters of deduction from true conclusions to true premises, and the resulting syntheses would be obvious. But Aristotle is clearly indicating here that the *possibility* of truth following from falsity is relevant to analysis after all, and it is for this reason that analysis is *not always* easy. Moreover, Aristotle says that mathematics uses convertible propositions more often than is the case in subjects treated by dialectic; he does not say mathematics always uses convertible propositions (as Robinson and Sayre would have it). In view of all this, it would seem that Aristotle recognized that analysis can require more intricate procedures than simply reversing the order of demonstration. Sometimes analysis is easy (when it is deductive) but sometimes it requires more ingenuity (or "quickness of wit [*agchinoia*]").[43]

Finally, the fact that Plato nowhere offers an explicit discussion of analysis (as opposed to dialectic) makes it virtually impossible to determine how Plato's thinking about analysis may have affected Aristotle. In particular, it is highly unlikely that the method Sayre ascribes to Plato can be used as a basis for interpreting Aristotle's understanding of analysis. Aristotle regarded non-convertibility as a difficulty for analysis, a difficulty which would be absent from the method Sayre attributes to Plato. In addition, Aristotle articulated several very serious criticisms of Plato's method of division[44]—which Sayre sees as integral to the "method of hypothesis."[45] There are several formidable obstacles, therefore, to the claim that Aristotle based his idea of analysis on a method in which division played so important a role,[46] even if it could be definitively shown that Plato had developed his "hypothetical method" from a deductive form of ancient geometrical analysis.

Thus, we are thrown back upon Aristotle's writings themselves to discover his own meaning of analysis. We therefore consider his own uses of the term, which in this chapter are divided into seven distinct but related groupings.[47]

C. SIMPLE REFERENCES TO THE *ANALYTICS*

We begin with the several passages where Aristotle's use of these terms merely refers to the text or science of the *Analytics* without giving any further hints as to its meaning. Thus, in *On Interpretation* (10 19b31), *Topics* (VIII.11 162a11, VIII.13 162b32), *On Sophistical Refutations* (2 165b9), *Metaphysics* (VII.12 1037b8), and *Nicomachean Ethics* (VI.3 1139b27–32) the terms *tois Analutikois* or *tōn Analutikōn* simply refer to the *Analytics* as a work.

Three related passages also refer, not to the text as such, but in a general way to its subject, analytics, in the same general, nonspecific fashion. *Posterior Analytics* mentions "that part of the analysis which concerns the [syllogistic] figures [*en tēi analusei tēi peri ta schemata*]" (II.5 91b13), that is to say, the early chapters of Book I of what we now call the *Prior Analytics*.[48] In the *Metaphysics* Aristotle complains of certain natural-science thinkers who "attempt to state how truth [of axioms] should be received show a lack of training in the analytics [*tōn analutikōn*]" (IV.3 1005b4).[49] Similarly, *Rhetoric* claims that "rhetoric is composed of the analytic science [*tēs analutikēs epistēmēs*] and of that branch of politics which is concerned with ethics" (I.4 1359b10). But the context of this passage reveals little more than the fact that rhetoric is directly concerned with the use of words [*logōn*] rather than with the things [*pragmatōn*] which are the objects of those words. By inference one may conclude that analytic science deals most directly with the usage of words. Beyond that, however, these general references reveal very little about the kind of "loosing up" we are to assign to Aristotle's idea of analysis.

D. DECOMPOSITION

There are four passages in Aristotle's *corpus* in which the nouns *analusis* and *analuseōs*, or various forms of the verb *analuein* are used with the simple reductionist meaning of a physical "breaking down," "decomposition," or "dissolution" into constituents. These are:

> Fire, air, water, earth, we assert, come-to-be from one another, and each of them exists potentially in each, as all things do that can be resolved [*analuontai*] into a common and ultimate substrate. (*Meteorology* I.3 339a36–b2)[50]

> Those [winds] which arise at the breaking up of a cloud and resolve [*analusin*] its density against themselves are called cloud-winds. (*On the Cosmos* 4 394b17–18)[51]

> By secretion or excretion I mean the residue of the nutriment, by waste-product that which is given off from the tissues by an unnatural decomposition [*analuseōs*]. (*GA* I.18 724b27–28)[52]

> Again, that which is subject to increase increases upon contact with a kindred body, which is resolved [*analuomenou*] into its matter, (*On the Heavens* I.3 270a22–23)[53]

In the last of these four passages "analysis" means "physical decomposition into constituents," its matter. In the context of this passage, Aristotle is arguing that the sort of body to which circular movement is natural is ungenerated, indestructible, and exempt from either increase or alteration (I.3 270a13–14). Aristotle's point in this passage is that which increases does so by incorporating more matter "from a kindred body" (i.e., composed of the same kind of matter) into itself, after first decomposing that other body into matter. Aristotle does not argue here why it is that these conditions cannot be fulfilled in the case of a naturally circulating body; he simply asserts it (I.3 270a24).[54]

There is also a fifth passage, similar to the foregoing, but whose interpretation is problematic:

> And it is plain that in all cases where a spermatic humour [waste-product] occurs this is also a secretion. This happens when it is dissolved [*analuētai*] into that which has come to it, just as when the coating falls away at once from stucco; for that which has come away is the same as that which was applied first. In the same way also the last secretion is the same as the first humour [waste-product]. (*GA* I.19 726b25–29)[55]

Some scholars have noted that it is out of place, while A. Platt finds it "totally unintelligible anywhere."[56] To my way of thinking, the term *analuētai* is indeed correctly rendered "dissolve," for when a second coat of plaster is applied to a first, it dissolves the first and thereby undoes its adhesion to the wall. Aristotle is speculating that something similar happens in certain kinds of seminal emissions.

The terms *analuontai*, *analusin*, *analusēos*, etc., in these five passages do indeed seem to describe a process of decomposition into constituents. Yet, the fact that fire, air, earth, and water are resolved into one another (339a36–b2)—not decomposed into a common, lower substance, a process Aristotle regarded as a physical (though not a conceptual) impossibility—is certainly at odds with the ordinary sense of decomposition.[57] Even here, then, Aristotle's notion of analysis cannot be adequately interpreted solely as a reduction of something to its underlying matter or constituents.

E. DISENTANGLEMENT

Much more revealing of Aristotle's meaning of analysis in relation to science are the following set of passages:

> Further, in most birds, the gut is thin, and simple when loosened out [*analuomenon*]. (*HA* II.17 509a17)[58]

> For in some animals [the gut] is uniform, when uncoiled [*analuomenon*], and alike throughout, while in others it differs in different portions. (*PA* III.14 675a33–34)[59]

> Indeed, men whose generative organs have been destroyed sometimes suffer from a looseness [*analuontai*] of the bowels caused by a residue which cannot be concocted and converted into semen being secreted into the intestine. (*GA* I.20 728a15–17)[60]

> Some of the women actually unwind [*analuousi*] the cocoons from these creatures [certain large larva], by reeling the thread off, and

> then weave a fabric from it; the first to do this weaving is said to have been a woman of Cos named Pamphile, daughter of Plates. (*HA* V.19 551b14–16)[61]

The last of these four passages reveals much about the other three. Although it speaks of Pamphile as having been the first to do the weaving, the context leads one to infer that she likely was also the first to discover how to do the unwinding; in any case, *someone* was the first to discover this. Moreover, the reference to the group of women[62] who are her successors further highlights the fact that a special knowledge and skill is involved in loosening the threads of the cocoon. While the larva Aristotle mentions is probably not the Chinese silkworm,[63] nevertheless it is likely that its threads were just as intricately and delicately intertwined. Thus, the skill of loosening, "analysing," the cocoon threads involves a careful disentangling, which in turn requires not just a knowledge of the cocoon's constituents (i.e., threads), but also an understanding of their way of being interconnected.

Likewise, the scientific examination of uncoiled intestines and stomachs implies a prior activity of uncoiling, since these kinds of intestines and stomachs are not found uncoiled in nature. (Even Aristotle's use of *analuontai* to refer to the condition of diarrhea reveals that, according to common opinion, an unnatural uncoiling of intestines was the root cause.) This uncoiling, analyzing, on the part of the investigating biologist requires for its accomplishment an understanding of the "secret of the compactness," so to speak, of these organs. The investigator must know—or find out—*how* these organs are coiled up. Otherwise, the effort to uncoil them will result in tearing and rupture, not uncoiling. The understanding of how intestines and stomachs are coiled up plays a role perfectly analogous to the knowledge required to disentangle the cocoons. Thus, knowledge of the manner of interconnection of constituents is at the heart of this, and other, types of analyzing. To analyze or "loosen up" in this sense, then, involves knowledge of the form of entanglement.

F. ANALYSIS AND THE FORMULA

We turn now to three passages where *analusis* and its cognates are used in connection with the definitions or formulae (*logoi*) of things. Two of these (*On the Heavens* III.1 300a11 and *On Generation and Corruption* II.1 329a23) pertain to a problem treated in Plato's *Timaeus* (47E–57D). In refining his "plausible account" of the Becoming of the Cosmos, Plato found it necessary to refine the common conception of the four elements—fire, water, air, and earth. Indeed, he even questioned whether these may properly be called elements, because of the fact that through heating, cooling, condensation, and rarefaction they can be transmuted into one another.[64] Plato continued, speculating that these four elements are constructed out of triangles ("planes" as Aristotle puts it)[65] assembled into the regular polyhedra. Thus, the four elements can be transmuted into one another as the polyhedra are resolved, analysed,[66] back into their constituent triangles and the triangles then reassembled back into other polyhedra.

Plato's discussion of the nature of the four elements, of course, is not to be taken at face value. Interpreting its meaning must begin with the fact that it is presented within the context of a "plausible account" that, in turn, is situated within the broader context of the dialogue. Aristotle was probably aware of this, since his criticisms are mostly directed toward a plural "they" (*On the Heavens* III.1 298b34, 300a7) who seem to have already taken it too literally. Aristotle states his criticism directly as follows:

> Nevertheless [Plato] carries his analysis [*analusis*] of the elements—solids though they are—back to planes, and it is impossible for the 'Nurse' (i.e., the primary matter) to be identical with planes.
>
> Our own doctrine is that although there is a matter of the perceptible bodies (a matter from which the so-called elements come to be), it has no separate existence but is always bound up with a contrariety. (*GC* III.1 329a23–27)[67]

Elsewhere he argues rigorously against the literal truth of the plausible account: Since planes (triangles) themselves are analyzable into lines, and lines into points, then if bodies, the elements earth, water, air, and fire, can be resolved into planes, they must be resolvable into points. But since bodies clearly have physical magnitudes such as weight and lightness, points and lines must also possess these magnitudes, which is contrary to the definition of a point.[68] The alternative is that

> Either there is no magnitude at all on their arguments, or magnitude can be annihilated, once granted that as the point is to the line, so the line is to the surface and the surface to the body; for all can be resolved [*analuomena*] into one another, and hence can be resolved [*analuthēsetai*] into the one that is primary [i.e., points], so that it would be possible for there to exist nothing but points, and no body at all. (*On the Heavens* IV.1 300a7–12)[69]

While it may seem that Aristotle is speaking here of analysis in the sense of physical decomposition into constituents (planes and points), more careful examination reveals something else. Aristotle does not say that the magnitudes of body and surface "are resolved" but "can be resolved" into points. What is going on instead, then, is a sort of conceptual or intellectual analysis. Aristotle's argument makes use of definitions, formulae, whose organization of terms into wholes[70] is meant to parallel the organization of real constituents into real wholes. Formulae are meant to express the "what-it-is" (*to ti esti*) of that which is defined.[71] Thus, the analysis of a regular polyhedron into triangles, or triangles into points, employs their definitions, their formulae. The formulae determine what the constituents are by determining the arrangement, the whole, the "what-it-is," which is being analyzed. To put it another way, polyhedra can be decomposed, cut up, or broken up in an indefinite number of ways, but their definitions are unique. It is the uniqueness and non-arbitrariness of these definitions that gives the force to Aristotle's refutations: once the analysis reaches the phrase, "without magnitude" in the formula of a point, one recognizes the impossibility of taking literally the "plausible account" of the *Timaeus*.

There is a comparable passage in the philosophical lexicon of the *Metaphysics*:

> [I]n formulae the first component, which is stated as part of the what-it-is [*to ti esti*], is the genus, and the qualities are said to be its differentiae . . . Things are called "generically different" whose immediate substrates [*prōton hupokeimenon*] are different and cannot be resolved [*analuetai*] one into the other or both into the same thing. *E.g.*, form and matter are generically different, and all things which belong to different categories of being; for some of the things of which being is predicated denote the what-it-is [*to ti esti*], others a quality, and others the various other things which have already been distinguished. For these also cannot be resolved [*analuetai*] either into each other or into any one thing. (V.28 1024b5–16)[72]

Once again Aristotle speaks of what "can (or cannot) be resolved" rather than what "is (or is not) resolved." The impossibility of resolving generically different things into one another (or some common thing) derives from their what-it-ises, which, in turn, are expressed in the formulae of their definitions. Whatever sort of analysis is possible in these cases is intellectual or conceptual, not physical. It proceeds through definitions to discover what the constituents are. When the most basic constituents [*prōton hupokeimenon*] of things—their genera—are found to differ, one realizes that there could be no physical analysis of one into another. Just as their definitions reveal the basic, underlying differences among their what-it-is (i.e., differences in their "first components," which are their genera), it is their ways of being constituted, their what-it-ises, that determine what the most basic constituent is. So once again, knowledge of formulae is essential to the intellectual analysis; and intellectual analysis reveals the possibility or impossibility of physical resolution.

In §E we saw that knowledge of the form of composition is essential to be able to loosen up or disentangle things without rupturing them. The present section has shown the important connection between the formula and analysis that can intellectually loosen up issues that are not clear at first glance. This all serves to underscore what will follow: For Aristotle there is a more important connection between analysis and knowledge of form than there is between analysis and matter.

G. ANALYSIS OF GEOMETRICAL FIGURES

In two places Aristotle speaks of analysis in reference to geometrical figures—*diagramma*. The first of these passages comes from the *Nicomachean Ethics*:

> We deliberate not about ends but about what contributes to ends. For a doctor does not deliberate whether he shall heal, nor an orator whether he shall convince, nor a statesman whether he shall produce law and order, nor does anyone else deliberate about his end. Having set the end they consider how and by what means it is to be attained; and if it seems to be produced by several means they consider by which it is most easily and best produced, while if it is achieved by one only they consider how it will be achieved by this and by what means *this* will be achieved, till they come to the first cause, which in the order of discovery [*ho en tēi heuresei*] is last. For the person who deliberates seems to inquire [*zētein*] and analyse [*analuein*] in the way described as though he were [analysing] a [geometrical] construction [*diagramma*] (not all inquiry appears to be deliberation—for instance mathematical inquiries—but all deliberation is inquiry), and what is last in the order of analysis [*analusei*] seems to be first in the order of becoming. (III.3 1112b12–24)[73]

The passage refers to geometrical investigations—called "*problēmata*" by Proclus[74]—which require for their solution the construction of a figure (*diagramma*) that meets certain stipulations: e.g., to bisect a given angle, to produce a square equal to a given rectangle, to cut a given line so that figures formed from its parts have specified relations to one another, and so on.[75] The famous gloss on Propositions XIII.1–5 of Euclid's *Elements* describes the geometrical "method of analysis" as a manner of proceeding which begins by assuming the figure as completed.[76] After assuming the sought-for figure as completed, there follows a process of resolving, of analyzing, which seeks a means of constructing the figure assumed-to-be-completed; then there follows a second stage of analysis to find a means of constructing the first means, and so on, until the "givens" of the problem—themselves also *diagramma*—are reached.

Let us first note that, in contrast to the account of Sayre, in Aristotle's account both the beginning point of geometrical analysis and its *terminus* is a *figure* [*diagramma*], and *not* a proposition or an equation.[77] Second, it might seem that this method of "assuming the figure as completed" and then working one's way back would be a fairly simple matter. However, when particular instances are examined, the method turns out to be considerably more complicated. If one adverts to cases drawn from ancient mathematical texts where such a task was actually undertaken, one will find that the problems are solved by embedding both the given and the sought-for figures within larger constructions whose relationships to the givens of the problem are neither obvious nor simple.[78]

Consider the example of a fairly basic problem from Euclid's *Elements*: "To construct a square equal to a given rectilinear figure" (Proposition II.14). Suppose that one takes the description of analysis literally and begins with the "sought as admitted"—in this case, a square already constructed which is equal to a rectangle, as in figure 1.1. In presupposing this square as "admitted," however, very little is achieved. Rather, the analyst must hit upon an ingenious construction, such as the one Euclid presents (see figure 1.2).[79] How does one move from assuming what is sought to a construction such as this in which the relationships among the given rectangle and the sought-for square are made manifest? This to my mind is the fundamental problematic of geometrical analysis.[80]

While the exact nature of this method of analysis requires a far longer discussion than is possible here,[81] a few points can be noted. First, experience in these matters reveals that there are commonly several ways to construct a required figure which initially suggest themselves to the analyst; however, these initial ideas seldom meet the exact stipulations of the problem. The correct constructions are usually more subtle. Second, what is being sought in geometrical analysis is not material elements (points, lines, figures), but a form of construction. One can easily draw a square, say, but to draw a square by means of a construction which correctly relates it to the given elements requires ingenious discovery.

Third, there is the question of just what Aristotle meant by that which "in the order of discovery/analysis is last." Clearly, Aristotle is proposing an analogy between ethical deliberation and geometrical analysis. It also seems evi-

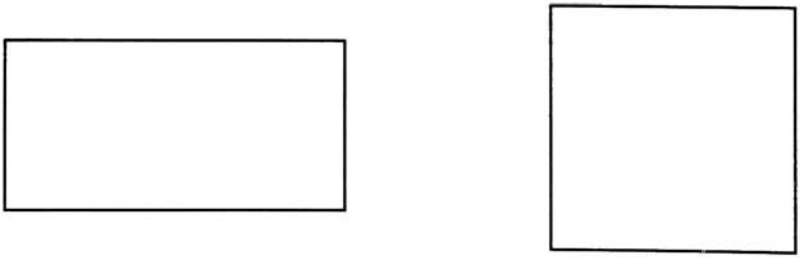

Figure 1.1

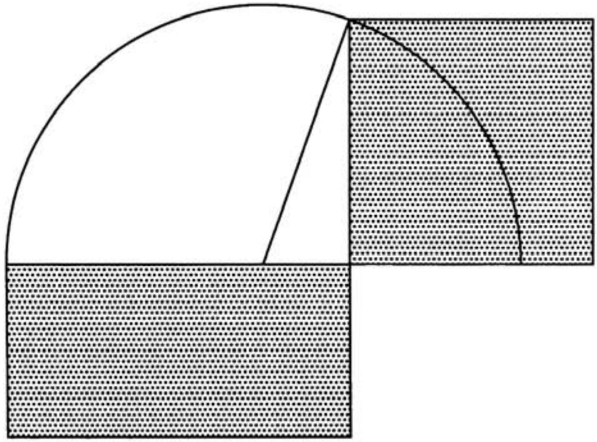

Figure 1.2

dent that Aristotle is attempting by this analogy to clarify the nature of deliberation on the basis of something more familiar—geometrical analysis.[82] What is first in the order of analysis are the "ends"—the geometrical figure to be constructed, and the ethical end sought in deliberation, respectively. Again, what is last in the order of geometrical analysis is the givens, which when they are finally fit into the encompassing construction, bring the analysis to completion. Analogously, what is last in the order of deliberation is the "given" means at one's disposal—one's already acquired capacities for producing (*poesis*, as in the case of the doctor producing health) and acting (*praxis*, as in the case of the agent acting virtuously). On the other hand, in the "order of becoming"—whether constructing or producing or acting—these givens are the point of departure, while their results come last.

Yet one cannot help but be struck by Aristotle's choice of language in this passage—he speaks of "the first cause" being last in the order of discovery. It does not seem likely that Aristotle would speak of a "diagram" as the first cause of anything. This raises a complex question of just what is the cause—and indeed the first cause—of facts; that issue will be addressed in detail in chapters 5 through 7. There it will be shown that, at least in the case of geometrical facts, that knowledge of causes is closely connected with discovery of figure constructions. But to anticipate, geometrical analysis of figures sets the stage for analysis leading to "first principles," i.e., principles "more intelligible in themselves."[83] For Aristotle, such principles have to do with form, not matter.[84] Form is what Aristotle meant by the "first cause" which scientific investigation comes to "last." What is ultimately "last" in the order of discovery (but "first" in the order of demonstration) are the principles and definitions, *not* the given points, lines, and figures of the problem, which might be regarded as "material" elements. In short, geometrical analysis is primarily concerned with the discovery first of constructed interconnections, and ultimately with intelligible form. The objective of geometrical analysis is *not* the reduction of something into its material elements.

Fourth, "next to last in the order of discovery" (so to speak) is *how* to rearrange and assemble the givens: to place the length and width of the rectangle end to end so as to make them the diameter of a circle; to insert the given angle into an isosceles triangle, and so on. This pattern of arrangement and assembly also is principally "formal," not material. One might argue that these rearranged (material) points and lines are what Aristotle means by the "first cause" the investigator comes to. To this I would respond that the rearranged points and lines come to be through the discovery of the "how," the forms, the ideas, of rearrangement. Without those forms, the rearranged points and lines *as* rearranged wouldn't exist. Thus, analysis of geometrical figures is a solving, a "loosing up," of what was merely a "given" array of elements into an intelligibly ordered arrangement—an intelligible arrangement which often brings delight and surprise, since its potential was initially unrecognized. Analysis in this sense, then, is more a matter of discovery of the forms of the construction and the forms of definition which underpin them, rather than of reduction to material elements.[85]

In speaking of analysis as a movement to the "first cause," Aristotle reveals the influence of discussions in Plato's Academy. Plato is said to have repeatedly reminded his students to be aware of whether they were "on the way to principles" (analysis) or proceeding from principles (synthesis) (*NE* I.4 1095a31–b2). Recent developments in mathematics—including the powerful method of analysis—were carefully studied and debated in the Academy. There, too, Plato stressed the value of excellence in mathematical thought as a prerequisite for philosophical thought, and this Platonic heritage[86] of Aristotle's idea of analysis also appears in *On Sophistical Refutations*:

> To take an argument in one's hand and see and solve [*lusai*] the fault in it is not the same thing as to be able to meet it promptly when one is being asked a question. For we often fail to recognize something which we know when it is presented in a different form. Furthermore, as in other spheres a greater degree of speed or slowness is rather a question of training, so in argument also; therefore, even though something may be clear to us, yet, if we lack practice, we often miss our opportunities. The same thing happens sometimes as with [geometrical] diagrams [*diagrammasin*]; for there we can analyse [*analusantes*] a figure, but not reconstruct [*suntheinai palin*] it; so too in refutations we know how the argument is strung together, but are at a loss how to take it to pieces. (16 175a21–31)[87]

Clearly "analyse" here also refers to the process of geometrical analysis. Presumably, the difficulty alluded to is that someone might know how to solve (that is, to analyze) a certain kind of geometrical problem, but not recognize a given problem as that kind of problem because of the manner in which it is posed. For example, one might know how to find the side of a square equal to a given rectangle, but not realize that this also solves the problem of finding the mean between the two sides of the rectangle.

This passage from *On Sophistical Refutations* is especially significant, for it indicates that Aristotle saw a crucial link between an approach to solving problems in geometry and an approach to solving problems in argumentation. Based upon what Aristotle wrote here and in other places to be discussed at length later in this book, I believe that influences from the method of geometrical analysis (which he probably first encountered in the Academy) can be detected in his method of analysis. I suggest there are signs of an extrapolation from the original field of analysis—i.e., geometrical applications—to a more general field—to the analysis of "what is said" in general, and to "what is said to be scientific" in particular. If my suggestion is correct, then it may be that Aristotle transformed the analytical method of geometry into a method of analyzing arguments. As we have seen, geometrical analysis involves not only the given and the sought-for lines, but also involves finding the figure which reveals the form of interconnection between what is given and what is sought. Along this line, as the passage indicates, Aristotle's method of analysis of arguments appears to involve not only reducing arguments to their underlying elements (premises), but also grasping how these given elements may be, not just arguments or words "strung together," but connected intelligibly into a logical argument—and possibly even, into a scientific demonstration. The remaining sections of this book will explore this suggestion in depth. (Of course in the case of the sophistical arguments which were Aristotle's primary concern in this passage, analysis will reveal that, in fact, they are not intelligibly connected, but merely "strung together.")

My suggestion here should not be taken as a rejection of the recent and extensive scholarly discussion regarding the historical development of Aristotle's "Apodeictics" and "Syllogistic."[88] While various scholars differ about the specific details of this line of development, I think we can at least be certain that Aristotle's earliest reflections on scientific demonstration ("Apodeictics") were preceded by and very likely grew out of still earlier reflections—recorded in the *Topics*—on the very general field of "sayings" (*logoi*) or "common places" (*topoi*) that attempt to prove or persuade. Again, there is consensus that the theory of syllogism recorded in the *Prior Analytics* was developed later than either the *Topics* or the theory of demonstration, and that the present *Posterior Analytics* contains the earlier "Apodeictics," at least partially modified in accordance with the theory of the syllogism.

I am substantially in agreement with this general scholarly opinion. Nevertheless, I maintain that while the working out of the standards for scientific demonstration ("Apodeictics") and "following of necessity" ("Syllogistic") may well have developed out of the reflections in the *Topics*, still Aristotle appears to have *organized* those results in accordance with the objectives of *analysis*. Thus, he likely drew upon the geometrical method of analysis as the paradigm for the more general, integrating context of his "Syllogistic" and his "Apodeictics." I further contend that this would be a perfectly natural development from the original investigation in the *Topics* which, as Irwin has noted, was a "handbook for the conduct" of a Socratic conversation.[89] Irwin goes on to point out that the intention of such conversations, especially for Aristotle, is to arrive at first principles; hence, one could say that such conversations are also concerned to "loose up" topics into their genuine "reasons why." Thus, Aristotle can be understood as bringing his innovations with respect to geometrical analysis, syllogistic analysis and his theory of demonstration to bear upon the tasks originally confronted by Socratic conversations. The relations among these themes will be explored in the remainder of this book.

H. ANALYSIS OF SORITES

Of the remaining passages from Aristotle's works where he uses *analusis* and its cognates, all are concerned with the analysis of arguments. Among these, the great preponderance are found in the *Prior Analytics* and pertain to the analysis of arguments into the three figures.[90] These topics will be treated in detail in the next two chapters. I offer one illustration here, however—the analysis of a "sorites"—as representative of the analysis of arguments, which is the subject matter of the *Analytics* itself.

There is a commonplace saying that the conclusion is "already contained" (perhaps implicitly) in the premises of an argument. For Aristotle, however, analysis of arguments reverses this commonplace: in some sense the conclusion "contains" the premises, and it is the process of analysis that makes explicit what the premises are.

Analysis is a matter of finding the intelligible interconnection among the constituents of something, of finding the form of something. Now the constituents of a conclusion most evident to us are its terms, *subject* and *predicate*. However, when the conclusion is merely stated, the intelligible connection among its terms is not at all evident. An argument is required to spell out this connection, because in an argument the premises are intelligibly interconnected and the terms of the conclusion occur in those premises. Thus, we may speak of an analysis or resolution of a conclusion to its premises. Moreover, we have seen that analysis can only take place in light of knowledge of the form of connection. In the present case, the relevant form of connection is among