Chapter 1

CONTINUING THE MATHEMATICAL PREPARATION OF MIDDLE-GRADE TEACHERS: AN INTRODUCTION

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What should a teacher know? Teacher knowledge has been the focus of much discussion in recent years. Shulman's (1986, 1987) term *pedagogical content knowledge* has gained broad acceptance as a way of distinguishing what good teachers know from what other knowledgeable people know. Although some (e.g., McEwan & Bull, 1991) argue that all scholarly knowledge is essentially pedagogic, we would agree with Shulman that scholarly knowledge of a content domain does not necessarily include knowledge of how to assist others in acquiring that knowledge. In fact, even the idea of pedagogical content knowledge does not adequately convey the full extent of what teachers must know, believe, and do to excel as teachers of mathematics. Harel (1993) has extended Shulman's ideas by suggesting that there are three components in a teacher's knowledge base: mathematics content, epistemology, and pedagogy, where *mathematics content* refers to the depth and breadth of the mathematics knowledge; *epistemology*, to the teachers' understanding of how students learn mathematics; and *pedagogy*, to the ability to teach in accordance with the nature of how students learn mathematics.

The reform movement in mathematics education "assumes that teachers will have a basic knowledge of mathematics, of pedagogical procedures, and of learners, and that they will apply that knowledge to the structuring of classroom learning activities for specific learners" (Center on Mathematics Teaching and Learning, 1990, p. 20). Yet indications from research studies with expert middle-grade teachers (e.g., Leinhardt & Smith, 1985) lead us to believe that few middle-grade teachers have this basic knowledge. This fact is not surprising, since most of these teachers have experienced only traditional classroom teaching, from either side of the desk, and have had little opportunity to explore the mathematics appropriate for middle-grade learners in any depth. With the exception of studies of researchers as teachers (e.g., Ball, 1993; Lampert, 1986; Mack, 1993), we have no studies of teachers, particularly at the middle-grade level, who *have* the strong content knowledge, understanding of how mathematics is learned, and exceptional pedagogical skills that can serve as witness to the basic assumption that these components are all necessary for the reform movement to succeed.

There is also relatively little information available on teacher change in the area of mathematics teaching. A few studies (e.g., Fennema, Franke, Carpenter, & Carey, 1993; Yackel, Cobb, & Wood, 1991) have been able to show that deep changes in teachers' understanding of the content and of students' learning of that content can have a major effect on how they structure their classroom learning experiences, but the studies have all taken place in the primary grades. Studies in the middle grades, such as those under the auspices of the Center for the Learning and Teaching of Elementary Subjects (Wiemers, 1990; Wilson, 1990) show how teachers struggle with calls for reform when they have little understanding of the mathematics, the values underlying the call for change, or what their students need to learn and how they can best learn it.

Several questions occur when we begin to explore the relationship between middle-grade teachers' understanding of the mathematics they teach and their instructional behaviors. What changes and shifts, both subtle and overt, can be noted in the way topics are treated by the teacher, as the teacher becomes more familiar with the mathematics involved, and comes to understand better how students learn this content? When a teacher has opportunities for study and reflection, how does her decision making change? What types of topics become more or less important to test? How do a teacher's priorities (in terms of time allocation, for example) change as she comes to better understand these topics herself? Finally, as a teacher's understanding of the mathematics of the middle grades develops, and as she becomes more aware of how students learn these concepts, how is student learning enhanced? The examination of this last question links the study of teaching to the study of learning. The difficulty of finding evidence of this link is recognized in the research literature. Silver (1985) noted that instructional studies characteristically failed to assess the direct effectiveness of the instruction. Brown (1993) also noted the difficulty of relating student outcomes directly to instruction and urged researchers to "make every effort to develop ways of evaluating the impact of instruction on students' understanding of the mathematics that reflect the goals of instruction" (p. 208).

At San Diego State University, we have been exploring these questions in a project funded by the National Center for Research in Mathematical Sciences Education (NCRMSE). This book grows out of the first two years of work undertaken to study the interaction between teacher knowledge and teacher decisions, and between teacher decisions and student learning, within the realm of number, quantity, and reasoning in middle-grade mathematics. The research is not complete, nor is much of it even reported here. And of course other mathematical topics are also appropriate for the middle grades—geometry, probability, and statistics are examples. We focus here, however, on the foundational mathematics middle-grade teachers need to know and the kind of instruction that takes place when teachers have this knowledge.

During the first year of the project, four teachers worked with the researchers to begin to clarify what it means to know mathematics and how this knowledge affects teaching. Our work with these teachers led to a set of case studies that appear in part 2 of this book; they are discussed later in this chapter. Armed with a better understanding of where we were heading, we began our second year with six middle-grade teachers and a goal of helping these teachers come to know the mathematics of the middle grades, and how their students come to be able to understand this mathematics and to reason quantitatively, proportionally, and multiplicatively. The teachers were all volunteers who felt a need to strengthen their mathematics. (One teacher was transferred and dropped out of the program; the other five continue to work with us.)

Over the course of ten months, the researchers and teachers met twice a month for three-hour seminars. Sometimes there were formal presentations; other times there were discussions of many sorts: of mathematical topics selected by the researchers or requested by the teachers; of the teachers' own thinking on items from a test of mathematical knowledge they had completed; of student performance and probable reasoning on items the researchers had designed to assess understanding of fractions and of proportions; and, of course, of the issues raised by the formal presentations and subsequent papers and of how to incorporate what we were all learning into unit and lesson planning and decision making. We observed each teacher several times during mathematics instruction in his/her classroom, and we continue to meet with the teachers and observe them this year. We are presently conducting case studies of these teachers as a means of answering our research questions.

It is the seminars we first want to share with the readers of this book. Since this research project was associated with the NCRMSE Working Group on Rational Number and Quantity in the Middle Grades, we had the good fortune of having access to other members of the working group to assist us with our seminars and serve as authors of the papers (now chapters). Each of the authors is well known for his or her research in the area of the mathematical ideas addressed in the paper, and each was asked to prepare a presentation and paper primarily based on personal research, "in a format accessible to teachers with some but not much background in mathematics and with some but not a lot of familiarity with research on teaching and learning." The collection of seminar papers that appears here served not only to inform the five teachers of this project; they form a basis for an initial content analysis appropriately undertaken by any teacher of middle-grade mathematics. By presenting their ideas first to a group of teachers, and then either writing or revising each paper in light of the seminar interactions, the authors have achieved a high degree of direct relevance to classroom teaching.

MATHEMATICS IN THE MIDDLE GRADES: OVERVIEW OF THE SEMINAR PAPERS

The seminars for the first half of the academic year focused on developing a deeper understanding of rational numbers, particularly in fraction form, and operations on these numbers. For the first presentation, *Instructing for Rational Number Sense* (see chapter 2), I intended my discussion of the types of instructional activities that lead to the development of rational number sense to set the stage for the forthcoming seminars of the semester. A fundamental theme of this chapter is that mathematically strong strategies for estimation and mental computation are idiosyncratic in nature, growing out of the properties of the numbers and operations involved, and that direct instruction on specific strategies is not sufficient to lead students to deal with number operations meaningfully. The motif that "mathematics must make sense" became evident throughout subsequent seminars, and the teachers often returned to this fundamental idea when they discussed their planning for instruction. The necessity of establishing benchmarks for fractions and decimal numbers to be used in estimating and making sense of solutions to problems involving operations on rational numbers was another theme that the teachers returned to several times.

Our first "outside" speaker was Thomas Kieren, whose presentation on *Creating Spaces for Learning Fractions* (chapter 3) was a wonderful example of how a researcher's knowledge of a particular content area, together with a strong desire to share that knowledge with teachers, can awaken in teachers a respect for the complexity of the topic and at the same time generate enthusiasm for undertaking truly meaningful instruction on that topic. In his chapter, Kieren sets out to create a space (opportunity, setting) for the reader to think about what is involved in making space for students to learn fractions. In the first part of the chapter he discusses the components of what teachers must know and be able to think about when planning instruction that opens up this space. The ideas in this section derive from research on fraction knowledge, his own and that of others, and provide the reader with much food for thought. In the second part of the chapter, Kieren provides two examples of fraction spaces that induce multiplicative and additive fractional thinking. All of the teachers used one or both of these activities during the following school year, and found that the activities did indeed open up students' thinking in ways that had never happened in their classrooms before.

In chapter 4, *Critical Ideas, Informal Knowledge, and Understanding Fractions,* Nancy Mack builds on her presentation to the teachers by identifying ideas critical to the development of understanding addition and subtraction of fractions: as a whole is divided into more parts, the parts become smaller; a fraction represents a single number; fractions can be represented in equivalent ways; and addition and subtraction require a common denominator. For Mack, these ideas can all be approached by drawing on students' informal knowledge, and she provides evidence for this claim through protocols from instructional sessions that were part of her own research. As our teachers came to understand the importance of these critical ideas as a foundation for instruction on addition and subtraction of fractions, they began planning their units with these ideas clearly in focus. Several times, in seminars, one or more of these critical ideas came up in the discussion.

After Mack's presentation, the teachers voiced their desire for a similar presentation focusing on multiplication and division of fractions. No one has explored these operations with fractions in the same way Mack has for addition and subtraction. However, Barbara Armstrong, one of the researchers on the project, together with Nadine Bezuk, had just prepared a set of activities for teaching multiplication and division of fractions for *Mathematics Teacher.* This preparation had involved reviewing research on this topic, their own and that of others, and long reflective discussions about the meaning of the operations and how they could be presented in real-life settings. Their presentation to the teachers covering their review of the research, their discussions, and finally the activities that grew out of their discussions led to chapter 5, *Multiplication and Division of Fractions: The Search for Meaning*. Multiplication and division of fractions were difficult, conceptually, for these teachers, as the reader can see in Armstrong and Bezuk's chapter, in Thompson's chapter (chapter 9), and again in the chapter describing the seminars (chapter 10).

The next two chapters move from a focus on fractions and fraction operations to a more general focus on issues of operating on rational numbers. Larry Sowder, in his chapter *Addressing the Story-Problem Problem,* laments the bad press story problems receive. It is no wonder story problems are dreaded, based on his review of the immature, inadequate, and ultimately unsuccessful strategies most students in his research used to solve story problems. He then argues that real-world actions can be related to math-world operations, and offers the reader an analysis of the types of real-world settings that lead to different mathematical interpretations of each of the operations. The focus is not on the story problem but rather on the situation and the manner in which it is represented mathematically. This linking between situations and operations offers the teacher a new way of thinking about story problems: It is not story problems that are important in and of themselves; they are important because they are the avenue leading to understanding of mathematical operations and the manner in which the operations link up to real-world situations.

Sowder refers in his chapter to the belief that "multiplication-makes-bigger; division-makes-smaller" as a phenomenon leading to many student errors. This "nonconservation of operation" is the taking-off point for Guershon Harel's chapter, *From Naive-Interpretist to Operation-Conserver.* Harel claims that the naive-interpretist student who solves rational number multiplication story problems by dividing in order to get a smaller number is in an unavoidable conceptual stage marking the transition from additive to multiplicative reasoning. It is not necessary, however, that children solve these problems using superficial considerations that lack meaning; they can take other approaches, additive in nature, until they have reached the stage where they can reason multiplicatively in such situations, as Harel shows through examples of student work. Harel's analysis of the transition from additive to multiplicative reasoning was perhaps the first time in our seminars that the teachers encountered ideas completely new to them.

Susan Lamon's chapter, *Ratio and Proportion: Elementary Didactical Phenomenology,* continues the exploration of the transition from additive to multiplicative reasoning, demonstrated by the ability to reason proportionally. She argues that the complex interaction of experiences that ultimately lead to proportional reasoning occurs over a long time period, and that therefore ratio and proportion should be considered as a unifying theme throughout the K–8 curriculum rather than as a single unit of study. Her own research has led her to uncover many "learning sites" critical to the development of proportional reasoning, and in her chapter she helps teachers become aware of these learning sites. She provides a wealth of activities designed to carry students through this development. Her suggestions and activities became a learning site for the project teachers. In chapter 10 we use the teachers' later discussion of some of her ideas as an example of how the seminars enriched the mathematical thinking of the teachers.

The final presentation was from Patrick Thompson, who spoke on *Notation, Convention, and Quantity in Elementary Mathematics,* three "often unmentioned" aspects of teaching for understanding. Thompson distinguishes quantitative operations from numerical operations; quantitative operations are conceptual and are used to imagine and reason about a situation. He claims that mathematics should help students see the world quantitatively, and he provides rich examples of what he means by this statement: examples that are complex either because they involve sophisticated ideas or because of the large number of relationships that need to be kept in mind simultaneously. The message of his chapter is that students need help in coming to understand that they must make sense of a situation before they can deal with the situation in a meaningful way.

Thompson's example of a situation that is complex because of the ideas involved (in this case, visualizing multiplication and division of fraction situations given a square divided into equal parts with some parts shaded) was very convincing to the teachers, whose struggle to visualize and to describe their visualizations is described in the first part of chapter 10, *The Role of Interaction in Promoting Teaching Growth.* The descriptions of seminars that form the first two parts of this chapter, together with final teacher comments on the seminars, are provided to give the reader a flavor of the interactions and discussions of the seminars and the benefits as viewed by the teachers. (We recommend that the reader read chapters 8 and 9 before reading chapter 10.)

We strongly believe that the ultimate influence of the ideas presented in the chapters in part 1 is highly dependent upon interaction among readers, such as might occur in a graduate class or in a teacher enhancement project. A quotation used in chapter 2 to indicate the importance of a classroom climate conducive to sensemaking is equally appropriate here:

> Environments that encourage questioning, evaluation, criticizing, and generally worrying knowledge, taking it as an object of

> thought, are believed to be fruitful breeding grounds for *restructuring*. . . . Change is more likely when one is required to explain, elaborate, or defend one's position to others, as well as to oneself; striving for an explanation often makes a learner integrate and elaborate knowledge in new ways. (Brown & Palincsar, 1989, p. 395)

THE VALUE OF CASE STUDIES: AN OVERVIEW OF PART 2

Kagan (1993) has described three uses for classroom cases: "as instructional tools to help novice teachers connect theory to practice and develop problem-solving skills; as raw data for research on teacher cognition; and as catalysts that can promote change in teachers' pedagogical beliefs and practices" (p. 704). While all three uses could be made of the case studies included in part 2 of this volume, it is the third use on which we focus here.

The four teachers who worked with the university researchers during the first year of the project were carefully selected; they had all been involved in prior research projects and/or graduate programs, all had been observed teaching, and all were known and highly respected by one or more of the researchers. While working with the four teachers on the research team, we, the university researchers, came to realize how much we were learning about teaching from these teachers, even though we ourselves all had prior school experience. These teachers proved to us that real instructional change is possible when teachers have the mathematical foundations coupled with the desire (in the case of these teachers, *passion* might be a better word) to undertake change. Hence, we decided to conduct case studies of the three middle-grade teachers in the group because we believed that others could also learn more about teaching mathematics from the study of these teachers' understandings, beliefs, and practices. (A paper focusing on the characteristics we found to be common to all the teachers is published elsewhere: Philipp, Sowder, Flores, & Schappelle, 1994.)

And we were correct; other teachers have found these case studies to be of value. We asked the teachers at a local school to read at least two case studies and reflect on them in terms of their value to them as teachers. It should be noted that the teachers at this school are in the process of curricular and instructional change in their mathematics program. They found that these case studies, with the descriptions of the classrooms and the comments of the teachers during interviews, validated their own attempts at change. Several were impressed by the gradual nature of change, and the length of time the case-study teachers had been involved in the change process; it helped to know that they could not be expected to change overnight. Reading the classroom protocols led them to want to teach like these teachers: "I need to do more of this"; "I would like to be able to facilitate this kind of discussion." But they also recognized their need for a deeper understanding of the mathematics involved: *"Teacher training is critical.* [I] need not *more* but *different* training than I had before." Though they at first felt overwhelmed by the experience and ability of the case-study teachers, the case-study teachers became "human" to them as they read the classroom descriptions and protocols: "Finally she becomes a real person for me to copy."

Although some of the teachers stated that they found the reviews of research literature at the beginning of the case studies, with the usual citation protocol, to be distracting from what they found to be the essence of the case studies (i.e., the descriptions of teachers and teaching), we have continued to use this format. Not only is the format acceptable within the research community, it is through these long introductions that we set the stage for the study of each teacher. One of the teachers did, in fact, comment on the usefulness of the discussion of the issues related to discourse in the classroom, and another commented on information she found of interest in a review regarding the effects of a strong background in mathematics.

We use the case studies to assist us in making points we believe to be important. In *Instructional Effects of Knowledge Of and About Mathematics,* a sixth-grade teacher, Phil, helps us make the argument that to be pedagogically effective, a teacher needs an understanding of the nature of mathematical knowledge and activity, in addition to a deep conceptual understanding of the mathematics involved. In *Orchestrating, Promoting, and Enhancing Mathematical Discourse in the Middle Grades,* we use several excerpts from Jo's class to clarify the role of discourse in promoting mathematical learning, and again make the point that this level of discourse is not possible if teachers do not have a sound grasp of the mathematics involved. In the third case study, *A Responsible Mathematics Teacher and the Choices She Makes,* we describe Betty's attempts, with her colleagues, to undertake reform on a schoolwide level at a restructured, inner-city school. One theme of the chapter is the necessity (and difficulty) of taking responsibility for one's students rather than blaming others for their problems and poor preparation, and what is required on the part of the teacher to take on this responsibility.

POSTSCRIPT

We do not wish to leave the reader with the impression that only the teachers benefited from these two years of project work. We, the researchers, came to a profound respect for the work and commitment of all the teachers with whom we worked. Through the project work, we, researchers and teachers, have acquired a common body of shared experiences and a common language with which to communicate; for example, we all now speak of the curriculum in terms of additive and multiplicative reasoning, and know we agree on what that means. We have all come to understand that teacher change at the middle-grade level is far more difficult than at the elementary level because of the focus on multiplicative topics in the middle grades and the difficulty teachers have in coming to understand these topics in the depth required for effective instruction. Teachers need time and opportunities such as these teachers have experienced. And finally, we have all come to *truly* appreciate the reality of the time required for real change, the frustrations and pain that teachers experience as part and parcel of this process of change, and the deep satisfaction teachers find along the way when their work leads to real and lasting understanding on the part of their students.

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