SAL RESTIVO

1

The Promethean Task of Bringing Mathematics to Earth

PROLOGUE

Whenever someone wants to give an example of a truth that is absolutely certain and indubitable, he or she is likely to use the Pythagorean theorem or a simple equation like 2 + 2 = 4. Martin Gardner (1981), for example, challenged the efforts of mathematicians such as Davis and Hersh (1981) and Kline (1980) to "undermine" the certainty of mathematics by giving the following example. In prehistoric times, "2 + 2 = 4" was "accurately modeled" whenever two dinosaurs met two dinosaurs in spite of the facts that there were no humans to observe the event and that the dinosaurs were incapable of comprehending or representing their gathering mathematically. This strategy appears across the entire range of cultural thought. The novelist Thomas Hardy wrote these words for Jude in *Jude the Obscure*:

> Is a woman a thinking unit at all, or a fraction always wanting its integer? How you argued that marriage was only a clumsy contract—which it is—how you showed all the objections to it—all the absurdities! If two and two made four when we were happy together, surely they make four now? I can't understand it, I repeat! (1969:370)

And the social theorist Karl Mannheim helped to keep a whole generation of sociologists of science outside the inner sanctum of "objectivity" when he wrote:

> Even a god could not formulate a proposition on historical subjects like $2 \times 2 = 4$, for what is intelligible in history can be formulated only with reference to problems and conceptual constructions which themselves arise in the flux of historical experience. (1936:79)

The mathematician, historian, and Marxist scholar Dirk Struik, a founder of the sociology of mathematics, referred to *both* the Pythagorean theorem *and* $2 \times 2 = 4$ in this defense of realism:

> Our conviction of the eternal validity of Pythagoras' theorem, of the fact that $2 \times 2 = 4$, is not based on some *a priori* conception, nor can it be shaken by any clever mathematician who in a big book with formulas concludes that these theorems are mere conventions. Our conviction is based on the fact that the theorems correspond to properties of the real world outside our consciousness which can be tested, and are accessible for testing to all persons from their earliest childhood. (1949:146–47)

But realism has not kept the nemesis of $2 + 2 = 4$, $2 + 2 = 5$, at bay. During the era of five-year plans in the Soviet Union, $2 + 2 = 5$ appeared. It was not designed as a serious threat to realism but rather to express the hope that the five-year goals might be achieved in four years. It is more interesting to see how the novelists Orwell and Dostoevsky used $2 + 2 = 4$ and $2 + 2 = 5$ to represent social and political systems, but in opposite ways. In Orwell's *1984*, O'Brien tells Winston that two and two are four *sometimes*: "Sometimes, Winston. Sometimes they are 5; sometimes they are 3; sometimes they are all of them at once. You must try harder. It is not easy to become sane" (1956:201). For Orwell, $2 + 2 = 4$ is a certainty against which to measure the totalitarian extremes of Big Brother, who is represented by $2 + 2 = 5$. Dostoevsky, on the other hand, uses $2 + 2 = 5$ in *Notes From Underground* to represent a challenge to rigid and routinized social and political realities:

> . . . twice-two-makes-four is not life, gentlemen. It is the beginning of death. Twice-two-makes-four is, in my humble opinion, nothing but a piece of impudence . . . a farcical, dressed up fellow who stands across your path with arms akimbo, and spits at

> you. Mind you, I quite agree that twice-two-makes-four is a most excellent thing: but if we are to give everything its due, then twice-two-makes-five is sometimes a most charming little thing too. (n.d.:139)

There is a noteworthy coincidence between this passage and Oswald Spengler's (1926:55–58) notion of number as an exemplar of the "become," the "hard set," and "Death." Spengler, a mathematics teacher, also offered a challenge to those who accepted the self-evidence of certain number facts:

> Even the most "self-evident" propositions of elementary arithmetic such as $2 \times 2 = 4$ become, when considered analytically, problems, and the solution of these problems was only made possible by deductions from the theory of aggregates, and is in many points still unaccomplished. (1926:84)

If there are readers who think these sorts of oppositions can only take place outside of mathematics proper, let them consider the following examples. Jourdain, for example, claimed that "Somebody might *think* that 2 and 2 are 5: we know by a process which rests on the laws of Logic [which refer to 'Truth'], that they make 4" But he almost immediately indicates that things may be more complicated. He claims that 1+1=2 may be "mistakenly written." This notation makes it look as if there are *two* whole classes of unit classes. In fact, there is only one, 1. 1 is a class of certain classes. Therefore, $1 + 1 = 2$ means that "if x and y are members of 1, and x differs from y, then x and y together make up a member of 2." (Jourdain, 1956:67–71).

Now consider that Bertrand Russell viewed the number 2 as "a metaphysical entity." But the class of couples, on the other hand, is indubitable and easily defined. It turns out, in fact, that the "class of all couples will *be* the number 2" (1956:542). So how self-evident is $1 + 1 = 2$? When Whitehead and Russell (1927) set out to prove this proposition, it took them almost eight hundred pages to establish the basis for the actual demonstration. The proof is reached nearly one hundred pages into volume 2 of *Principia Mathematica* ($1 + 1 = 2$ is theorem #110.643). It took Leibniz, incidentally, only six short lines to prove $2 + 2 = 4$. He considered $1 + 1 = 2$ a statement of pure mathematics, true as a consequence of the law of contradiction and therefore true in all possible worlds.

For Plato, $1 + 1 = 2$ describes relations that do not change between objects that do not change. It is independent of any preliminary constructive act; it reflects the reality of the Forms. Aristotle, by contrast, considered mathematics to be about idealizations that mathematicians construct (on the social contexts of these differences, see Restivo, 1983). For Kant, 1 + 1 = 2 is a synthetic proposition and a priori. Later the logicists, formalists, and intuitionists would offer mathematical arguments for different conceptions of 1 + 1 = 2. For the logicists, 1 + 1 = 2 can be expressed in terms of the logic of truth functions, the logic of quantification (in particular the concept of 'universal quantifier'), and the logic of classes (especially the concepts of 'sum class' and 'product class'). The statement "1 apple and 1 apple make 2 apples" is, just like 1 + 1 = 2, a statement of logic; it is not an empirical statement about *this* world, but rather a statement about "classes of classes in particular" (Korner, 1962:53).

For the formalists, 1 + 1 = 2 is an object and not a statement. Thus, as an object it is neither true nor false. But when it gets labeled in *their* reality as a "theorem-formula," the labeling can be viewed as a "true-or-false" phenomenon.

The intuitionists conceive 1 + 1 = 2 and one apple and one apple make two apples as "exact characteristics of self-evident, intuitive constructions" (Korner, 1962:178). It should be clear, then, that everyday arithmetic is not a simple matter for philosophers and mathematicians!

If we broaden our perspective somewhat, and at the same time leave the heights of *Principia Mathematica* and come down to a world of tuna fish, rocks, and cows, we can see why the apparently simple procedure of *adding* is *empirically* problematic. Consider the following problems from Davis and Hersh (1981:71–74; and see Hogben, 1940:32–34):

1. One can of tuna fish costs $1.05; how much do two cans of tuna fish cost?
2. A billion barrels of oil costs x dollars; how much will you have to pay for a trillion barrels of oil?
3. A banker computes your credit rating by allowing 2 points if you own your house, and then adds 1 point if you earn over $20,000 a year, and 1 point if you have not moved in the last five years; the banker subtracts 1 point if you have a criminal record, and 1 point if you are under 25, and so on. What does the final sum mean?
4. On an intelligence test, you get 1 point if you know George Washington was the first U.S. president; another point if you know some fact about polar bears, another point if you know

about Daylight Saving Time, and so on. What does the final sum mean?

5. One cup of milk is added to one cup of popcorn. How many cups of the mixture will result?
6. One person can paint a room in one day. He/she is joined by another person who can paint a room in two days. How long will it take for the two of them to paint a room?
7. I have one rock that weighs one pound, and I find a second rock that weighs two pounds, How much will the two rocks weigh together?

These questions cannot be answered by simply following the imperative, Go ahead and add. You must know about the relationship between summing and discounting, dealing with a diminishing resource, "figures of merit," measurement problems, and so on. In problem 5, since a cup of popcorn can very nearly absorb a cup of milk without spilling, we could represent this as $1 + 1 = 1$. And in problem 7, we need to consider that weighing two rocks together can bring into play nonlinear spring displacements.

Kline (1962:579–83) points out that we can object to the "truth" of $2 + 2 = 4$ on the grounds that the associative axiom is based on limited experience. But he notes further that problems such as those listed above show that there are even weaker links between arithmetic and "the real world." Consider, for example, the effect of supply and demand on the price of two herds of cattle sold separately and together; or the relationship between what is arithmetically correct (for example $2 \times 1/2 = 1$) versus what the case is in a particular instance (for example, do two half-sheets of paper make one whole sheet?); or adding forces that act at right angles to each other (in which case, for example, we could find that $4 + 3 = 5$). Kline's conclusion is not really the relativism his critic Gardner claims (see p. 3). His conclusion is that the system of $2 + 2 = 4$ arithmetic is based on limited and selected experiences. Ordinary arithmetic fails to describe correctly the results of what happens when gases combine by volume or one crop combines with another or when one cloud combines with another.

There *are*, in fact, special arithmetics for dealing with special situations. Clocks that use the numerals 1 to 12 operate according to a modular arithmetic in which, for example, $10 + 6 = 4$. A finite group defined by its multiplication table according to a famous dictum by Cayley manifests the associative law but not the commutative law. Hamming (1980:89) uses an arithmetic and algebra in which $1 + 1 = 0$ (conventional integers are used as labels, and the real numbers are used as probabilities). In group theory, a set may be Abelian or non-Abelian according to whether the combining rule is commutative or non-commutative (cf. Wilder, 1981:39–40; Scriba, 1968:7).

We have not yet really reached the stage of an explicit social theory of mathematics, and yet there is plenty of reason already to ask questions about self-evidence. We have been alerted, however, to logic and self-evidence as cultural resources that can be used to defend or attack a social order (by Orwell and Dostoevsky, for example). Consider, furthermore, I. C. Jarvie's (1975) comment that nothing we would want to call mathematics or morality can be "localized;" there cannot be culture-bound answers on the question of whether children should be tortured or whether mathematical propositions are true or false. This explicit juxtaposition of mathematical and moral certainty is quite interesting. Let's see what happens when we try to put all of this into a sociological framework.

Mary Douglas writes that a self-evident statement is a statement "which carries its evidence within itself. It is true by virtue of the meaning of the words" (1975:277). Douglas takes her examples from Quine's discussion of self-evident sentences such as "all bachelors are unmarried men" and (no surprise here!) 2 + 2 = 4. Such sentences, Quine contends, "have a feel that everyone appreciates." People react to denials of such sentences the way they react to "ungrasped foreign sentences." Quine concludes that if analyticity intuitions operate substantially as he suggests they do, then "they will in general tend to set in where bewilderment sets in as to what the man who denies the sentence can be talking about" (Quine, 1960:66–67). Douglas improved Quine's account. Between the psychology of the individual and the public use of language, she inserted a dimension of social behavior in which logical relations also apply:

> Persons are included in or excluded from a given class, classes are ranked, parts are related to wholes . . . the intuition of the logic of these social experiences is the basis for finding the *a priori* in nature. The pattern of social relations is fraught with emotional power; great stakes are invested in their permanence by some, and then overthrown by others. This is the level of experience at which the gut reaction of bewilderment at an unintelligible sentence is strengthened by potential fury, shock and loathing. (Douglas, 1975:280)

The reason some of us can be so furious in identifying and opposing the illogical is that it is a threat to a *moral* order.

This prologue takes us to the threshold of a secular, earthbound view of mathematics as a social and cultural construct, product, and resource. This book is a contribution to an unfolding story of mathematics that is stripping it of the last clinging vestiges of Platonism and related forms of idealism. This book is not the end of the story, by any means. As readers will see, Platonism is not easy to uproot, even when the Platonist decides it is a good idea to embrace the idea of mathematics as a social practice. Nonetheless, this book is an important step forward. It provides some basic resources not only for grounding mathematical knowledge in mathematical practice but perhaps more importantly for linking it to the problem of improving the conditions under which we teach and learn mathematics and more generally the conditions under which we live.

INTRODUCTION

From at least the time of Plato, it has been customary to write stories about mathematics as if mathematics had fallen from the sky. This volume is a contribution to hauling "this lofty domain from the Olympian heights of pure mind to the common pastures where human beings toil and sweat" (Struik, 1986:280). The authors are all concerned with the bearing of mathematical practice on the production or construction of mathematics. But this volume does not outline a monolithic program. It is a portrait of struggles—with the ghost of Plato, for example, or with the spectre of mathematical practice. It is the editors' contention that these struggles are the starting point for the further development of a new understanding of mathematics already abroad, one that is grounded in social realities rather than metaphysical and psychological fictions. We therefore run the gamut from philosopher Michael Resnik's defense of a form of Platonism to my own sociological assault on philosophy and epistemology of mathematics. But we are not simply concerned with exploring a narrow band of philosophy and sociology of mathematics here. A great deal of space is devoted to issues of politics and values in mathematics and mathematics education.

Three general forms of mathematical studies are exhibited here: philosophy of mathematics, political and social theory of mathematics and mathematics education, and sociology and sociological history of mathematics. The opportunity for this undertaking was provided by the publication in 1988 and 1989 of special issues of two journals, *Philosophica* (edited by one of my coeditors for this volume, Jean Paul

Van Bendegem) and *Zentralblatt für Didaktik der Mathematik (ZDM)*, guest edited by my other coeditor, Roland Fischer). Van Bendegem brought together a group of students of mathematics, all of whom agreed about the need to pay attention to what real mathematicians can and actually do. We have selected four of the *Philosophica* contributions to publish in this volume.

Fischer called on a more diverse group of researchers and educators to assemble his two *ZDM* issues. These contributors were asked to address problems in the politics of mathematics education. Here, too, we find a concern for grounding our understanding of mathematics in mathematical practice. Selections from the *ZDM* issues appear in parts 3 and 4 of this volume. A number of these articles were originally published in German, and we are pleased to be able to publish them here for the first time in English translations.

Part 4 begins with a chapter by Fischer on mathematics and social change. This chapter reflects Fischer's editorial concerns in putting together the *ZDM* issues. The second chapter by Mehrtens is an English version of the paper he published in *ZDM* on the sociological history of mathematics under national socialism. The concluding chapter is my final word on the sociology of mathematics. This and the introduction to this volume are based on my contributions to the special issues of *ZDM* and *Philosophica*.

In the following pages, I briefly introduce the contributions to this volume and show how and to what extent they deal with the social realities of mathematical practice.

PHILOSOPHICAL PERSPECTIVES

The section on philosophical perspectives is introduced by coeditor Jean Paul Van Bendegem's paper. Just as Karl Marx demanded that social thinkers start their inquiries by looking at the real everyday activities of real people, so Van Bendegem argues that the study of mathematics must be grounded in mathematical practice—in particular, a theory or model of mathematical practice. Van Bendegem represents a small but growing group of philosophers of mathematics who are challenging traditional Platonist assumptions by asking questions such as, What are mathematicians really like, what *can* they really do, and what *do* they really do?

The fact is that pitting the spectre of mathematical practice against Platonism has not cleanly and quickly vanquished Platonism. To illustrate this point, we have included a paper by Michael Resnik, who advocates a form of Platonism. He finds himself in the position of having to confront the spectre of mathematical practice abroad in his field, but he meets the challenge not with a sociological tool kit and agenda but with a naturalistic strategy. Resnik's paper helps us to recognize the unresolved controversies alive in contemporary math studies. Even Van Bendegem, it should be noted, seems intent on preserving the possibility of something we could call "*the* mathematics." And while he wants to sociologize our understanding of mathematics, he envisages doing this in the form of theorems.

Resnik's goal is to show how a postulational account of mathematical knowledge could count as a naturalized epistemology. Here he is indebted to Quine's contention that epistemology is really a chapter in psychology and therefore a piece of natural science. Resnik appreciates the need to explain critically how we come to accept mathematics in terms of the everyday practices of mathematicians. His strategy is, however, not to sociologize mathematics but rather to naturalize Platonism. In the final analysis, his goal is to make pure mathematics a part of natural science.

Whereas Resnik addresses the increasingly obvious need for philosophers of mathematics to pay attention to mathematical practice, Thomas Tymoczko seeks to make mathematical practice the *focus* of the philosophy of mathematics. The major influence we see at work here is the quasi-empiricist imperative in the works of Imre Lakatos and Hilary Putnam. Quasi-empiricism involves an emphasis on mathematical practice and an orientation to mathematical methods as scientific methods. This approach links mathematics to scientific realism. In this view, mathematical objects, like the objects of scientific study, are considered "real." And mathematicians, like scientists, are considered to be discoverers—they make their discoveries in the realm of mathematical reality.

The emphasis on mathematical practice is designed in part to free philosophy of mathematics from all forms of foundational programs. But philosophers of mathematics who take mathematical practice into account are not unified on the questions of realism and Platonism.

Tymoczko's aim is to sort out some of the differences among philosophers of mathematics who come under the banner of quasi-empiricism. He focuses on contemporary set theory, treating it as one branch of mathematical practice rather than as a foundational program. He argues against adopting scientific realism in this case and suggests that perhaps some form of Putnam's "internal realism" is applicable. According to Tymoczko, there is no transfinite world "out there" waiting to be discovered. He stops short of the sort of analysis that would make sociological sense out of the concept of 'internal realism.'

Yehuda Rav describes the philosophy of mathematics as the study of the nature of mathematics, its methodological problems, its relation to reality, and its applicability. He notes the marked drift in the current literature towards an analysis of mathematical practice. He applauds this as a way to liberate philosophy of mathematics from Platonism, logicism, intuitionism, and formalism. Rav explores evolutionary epistemology as an approach that would ground mathematical studies in practice and simultaneously keep these studies abreast of the latest developments in the philosophy of science. Evolutionary epistemology, according to Rav, offers us a way to escape the "quicksand of neo-scholasticism and its offshoots."

Following Donald Campbell, Rav argues that, minimally, an evolutionary epistemology would be an epistemology that recognizes and is compatible with the biological and social evolution of humans. He argues furthermore that evolution—even biological evolution—is a knowledge process. The natural selection paradigm for knowledge evolution can be generalized to the cases of learning, thought, and science and also to mathematics. Rav argues, finally, that Platonism is completely incompatible with evolutionary epistemology.

This first set of papers illustrates that the spectre of mathematical practice is abroad in the philosophy of mathematics, but that it has not yet vanquished the ghosts of foundationalism and Platonism. And, so far, the efforts in this direction seem generally to lead to forms of naturalism rather than to sociological theories. In part 3, we explore approaches to mathematics studies that still fall short of radically sociologizing mathematics but that are more firmly grounded in the politics of mathematics and mathematics education.

MATHEMATICS, POLITICS, AND PEDAGOGY

Part 3 is introduced by coeditor Roland Fischer's paper on mathematics as a means and as a system. Set theory plays a prominent role in his argument. Set theory, Fischer claims, does not allow us to define elements in a set in terms of their relationships with all the other elements in a set. This notion, he argues, is mirrored in "rational man" economic theory. He then points out the crucial flaw in "rational man" theories: the failure to recognize a fundamental sociological thesis propounded by Karl Marx that the individual is really an ensemble of social relationships.

Fischer defines a field called "didactics of mathematics," a collective effort to study and shape the relationship between human beings (individuals, groups, and whole societies) and mathematics. Fischer views mathematics as a *means* for people, a resource, *and* a *system* of concepts, algorithms, and rules embodied in ourselves, our thinking, and our actions. Fischer pursues these themes further in the first chapter of part 4. Here he makes quite clear his humanistic concerns, and the importance of collective self-reflection for improving the human condition. He argues that mathematics is a mirror of humanity and can contribute to the process of collective self-reflection. He stresses the importance of valuing freedom, autonomy, flexibility, and playfulness in social affairs and mathematics. Given its potential as a factor in and for humanistic social changes, mathematics should not be permitted to be totally focused on its traditional tasks.

Helga Jungwirth's paper illustrates the inability of prevailing approaches and models in math studies to reconstruct "mathematical cultures" in ways that reveal the contexts of our relations to mathematics. Her objective is to identify the underlying assumptions and limitations of research on women and mathematics. She argues that these assumptions and limitations reflect the lack of systemic-ecological contextualist models in math studies. The result is a correspondence between claims about discovering the specific relations between women and math, the goal of changing those relations, a psychological approach to the problem, and the use of empirical-analytical methods. Jungwirth claims that the social realities of mathematical practice are overlooked, and that stereotypical thinking about gender as well as an uncritical view of mathematics dominate the conventional paradigm. In order to see what sorts of changes we need to look toward to solve the problems Jungwirth identifies, we should look to papers like the next one by Nel Noddings.

Noddings argues forcefully that mathematics classrooms should be politicized. She integrates Paolo Friere's pedagogy and a feminist perspective to ground her contention that students should be involved in planning, challenging, negotiating, and evaluating the work they do in learning mathematics. Such involvement, in fact, can be expected to facilitate math learning.

One of the core approaches to contemporary math pedagogy is constructivism. Constructivists in math education argue that all mental acts (perceptual and cognitive) are constructed (this is not the same sort of constructivism found in the social problems literature or in social studies of scientific knowledge). Noddings's argument is that constructivism, assuming it is a viable pedagogy (an assumption coming under increasing scrutiny and criticism), must be embedded in an ethical or political framework if it is going to have an effect on reforming classroom practice.

Noddings links what goes on in the mathematics classroom to education for civic life in a free society. She is therefore necessarily critical not only of math pedagogy but of modes of schooling in general. The primary aim of every teacher must be not simply to promote some uncritical notion of achievement but the growth of students as self-affirming and responsible people.

The theme of political dimensions in mathematics education is pursued further by Ole Skovsmose. He criticizes the focus of mathematics pedagogy on the individual learner and is thus necessarily critical of Piaget's genetic epistemology (widely influential among science educators). Like Noddings, Skovsmose wants us to explore the epistemological potential of the relationships between the children in a classroom. Skovsmose's paper illustrates that one of the paths to a focus on social interaction and social practice starts from Wittgenstein's work in the philosophy of language and Austin's theory of speech acts.

In the end Skovsmose asks us to rethink the concept of 'knowledge' so that Platonic dreams are replaced by realistic ideas about knowledge conflicts and reflexivity that make it possible to evaluate technologies.

The final contributor to part 3 is Philip Davis, whose work combines the premise that mathematics is social practice, the imperative that mathematical practice be made the object of our descriptive and interpretive strategies, and the assumption that we live—and must learn to live—in a mathematized world. Mathematical education must, Davis argues, be reoriented to help us find the right ways to understand mathematical practice.

MATHEMATICS, SOCIETY AND SOCIAL CHANGE

In the lead article for part 4, coeditor Roland Fischer reinforces some of the ideas introduced in part 3 by arguing for an orientation to the tasks that face humanity today, the importance of a strategy of collective self-reflection for undertaking those tasks, and the role of mathematics in fostering self-reflection. Fischer draws on the ideas of the sociologist Niklas Luhmann, who has dealt with the problem of complexity from an epistemological standpoint. Fischer is concerned to show how mathematics can help us deal with establishing and understanding the complex interconnections that characterize our world. He also stresses that mathematics itself is an expression of social relations, an idea developed more fully in my concluding essay in this volume.

The second paper in part 4, by Herbert Mehrtens, in an example of a sociologically grounded history of mathematics. Luhmann's work once again enters the picture by providing Mehrtens with a conception of 'social system.' Mehrtens's paper is a survey of problems arising in German mathematics under national socialism. He begins by describing the social system of mathematics in Germany during the rise and rule of national socialism, and the characteristics of national socialism. He then analyzes the background and structure of the attempt to construct a "German" mathematics related to Nazi ideology. Mehrtens goes on to examine the basic relations between mathematical and political thought. The focus here is on the twin processes of adaptation and resistance by professional societies. Mehrtens argues that social differentiation within the system of mathematics, as well as its modern cognitive and social universality, were preconditions of adaptations.

In the concluding chapter in part 4, I outline a radically sociological approach to thinking about mathematics. The approach is radical because it claims complete jurisdiction over the problems of the nature of mathematics and mathematical knowledge for sociology. It is not my intention to simply be political. The rationale for such an approach has been developing since the 1850s. Once we realized that individual human beings are in fact *social* beings, it was only a matter of time before it occurred to some social theorists that the mind and thinking are social phenomena. The upshot of this sociological revolution is the argument and sketch in the concluding essay.

CONCLUSION

The idea that mathematics, or any other form of knowledge, falls from the sky is quickly fading. But sometimes, as in the case of our ideas about the gods, the difficulty of coming up with a satisfactory alternative explanation keeps the old idea alive. That is the case in mathematical studies today. Some students of mathematics recognize that there has been progress in social studies of mathematics, and they take up the rhetoric of culture, communication, and community in their analyses of mathematics. Others adapt a fashionable (for good reasons, let me add) interdisciplinary approach, but play this game close to the chest so that traditional disciplinary boundaries and assumptions are not seriously threatened. Others rush into social constructivism with such abandon that they are almost at a loss for words when it comes to exchanges with less adventurous colleagues. But within all of this diversity, there is a growing awareness, if not of a theoretical social constructivism, at least of the necessity of attending to the social practices that people engage in to produce or construct knowledge and facts, including mathematical knowledge and facts.

We are in the early stages of transforming insights from the works of Durkheim, Neitzsche, and others into collective representations. This volume is one small step in support of that transformation.

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